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Analysis of ordinary and radiative muon capture in liquid hydrogen

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The experimental data on the ordinary muon capture (OMC) rate and the partial branching ratio of radiative muon capture (RMC) in liquid hydrogen are simultaneously analyzed using theoretical estimates for the atomic OMC rates obtained in chiral perturbation theory using the most recent physical constants. Since both data have been obtained in a liquid hydrogen target, we reexamine the formulas for relating the atomic OMC and RMC rates to the liquid hydrogen OMC and RMC rates, respectively. Although the analysis is influenced significantly by the ambiguity in the molecular state population, we can conclude that, while the OMC data can be reproduced, it is not possible to explain the OMC and RMC data simultaneously.

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1. Introduction

Ordinary and radiative muon capture (OMC and RMC) on a proton

$$\mu^- + p \to n + \nu_\mu, \qquad \text{(OMC)} \tag{1}$$
$$\mu^- + p \to n + \nu_\mu + \gamma, \quad \text{(RMC)} \tag{2}$$

are fundamental weak-interaction processes in nuclear physics and a primary source of information on g_P, the induced pseudoscalar coupling constant of the weak nucleon current, see *e.g.* [1, 2]. The most accurate existing measurements of the OMC and RMC rates have been carried out using a liquid hydrogen target, which unfortunately makes the analysis of the data sensitive to the molecular transition rates in liquid hydrogen. We denote by Λ_{liq} the OMC rate in liquid hydrogen. The experimental value obtained by Bardin *et al.*[3] is

$$\Lambda_{liq}^{exp} = 460 \pm 20 \quad [\mathrm{s}^{-1}] \qquad \text{(OMC)}. \tag{3}$$

As for RMC, Jonkmans *et al.*[4] measured the absolute photon spectrum for $E_\gamma \geq 60$ MeV and deduced therefrom the partial RMC branching ratio, R_γ, which is the number of RMC events (per stopped muon) producing a photon with $E_\gamma \geq 60$ MeV. The measured value of R_γ is [4, 5]

$$R_\gamma^{exp} = (2.10 \pm 0.22) \times 10^{-8} \quad \text{(RMC)}. \tag{4}$$

Surprisingly, the value of g_P deduced in [4, 5] from the RMC data is ~1.5 times larger than the PCAC prediction [6]. By contrast, the value of g_P deduced in [7] from the OMC data is in good agreement with the PCAC prediction.

On the theoretical side, the early estimation of g_P was made using PCAC. Heavy-baryon chiral perturbation theory (HBχPT), a low-energy effective theory of QCD, allows us to go beyond the PCAC approach, but the results of detailed HBχPT calculations [8] up to next-to-next-to-leading order (NNLO) essentially agree with those obtained in the PCAC approach. Thus the theoretical framework for estimating g_P is robust. The key quantities in analyzing OMC and RMC are the atomic rates, Λ_s and Λ_t, where Λ_s (Λ_t) is the capture rate for the hyperfine singlet (triplet) state of the μ-p atom. The atomic rates of OMC and RMC have also been estimated in the framework of HBχPT [9, 10, 11, 12, 13, 14, 15]. The expressions obtained in HBχPT have been found to be essentially in agreement with those of the earlier work [16, 17, 18, 19, 20]. It has also been confirmed that the chiral expansion converges rapidly, rendering estimates of the OMC and RMC rates obtained in χPT extremely robust. As for the numerical results, however, the earlier estimates of the atomic OMC rates, *e.g.* [16, 17], need to be revised because some values of the input parameters (g_A, $f_{\pi NN}$, *etc.*) used in those estimates are now obsolete. In Ref. [10], we provided updated estimates of Λ_s and Λ_t based on HBχPT (up to NNLO). A notable finding in [10] is that the use of the recent larger value of the Gamow-Teller coupling constant, g_A, gives a value of Λ_s that is significantly larger than the older value commonly quoted in the literature, see Refs.[10, 15].

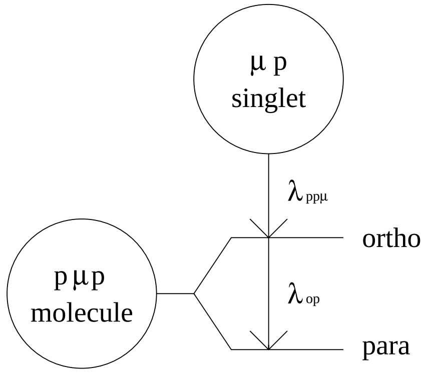

Figure 1: Atomic and molecular states relevant to muon capture in liquid hydrogen; $\lambda_{pp\mu}$ is the transition rate from the atomic singlet state to the ortho p-μ-p molecular state, and λ_{op} is that from the ortho to para molecular state.

To make comparison between theory and experiment, one needs to relate the theoretically calculated atomic OMC and RMC rates to Λ_{liq} and R_γ, respectively. For convenience, we refer to this relation as the A-L (atom-liquid) formula. Bakalov *et al.* [21] made a detailed study of the A-L formula, and they gave an explicit expression for Λ_{liq} (see Eq. (56c) in Ref. [21]). In our previous work [10] we analyzed Λ_{liq} using the A-L formula of Bakalov *et al.* and found that the best available estimates of the atomic capture rates based on HBχPT would lead to a value of Λ_{liq} that was significantly larger than Λ_{liq}^{exp}. We also reported that, by introducing a molecular state mixing parameter, ξ, considered by Weinberg [22], it was possible to reproduce Λ_{liq}^{exp} and R_γ^{exp} simultaneously. However, the A-L formula of Bakalov *et al.* does not correspond to the experimental condition of OMC; to compare with Λ_{liq}^{exp}, the time sequence of the experimental measurement should be considered [3].

In this work we reexamine Λ_{liq} and R_γ by incorporating the experimental conditions as well as the updated physical constants into our analysis. In particular, we investigate the influence of the ambiguity in the transition rate between the molecular states (to be discussed below); we also examine the dependence of the results on the values of g_P and the molecular parameter ξ (see below).

2. Muonic states in liquid hydrogen

To evaluate Λ_{liq} and R_γ from the calculated atomic OMC and RMC rates, we need to know the temporal behavior of each of the various μ-capture components (capture from the atomic states and capture from p-μ-p molecular states). Fig. 1 schematically depicts various competing A-L processes. A muon stopped in liquid hydrogen quickly forms a muonic atom (μ-p) in the lowest Bohr state. The atomic hyperfine-triplet state (S=1) decays extremely rapidly to the singlet state (S=0), with a transition rate $\lambda_{10} \simeq 1.7\times10^{10}$ s^{-1}. In the liquid hydrogen target a muonic atom and a hydrogen molecule collide with each other and form a p-μ-p molecule with the molecule predominantly in its ortho state.

We denote by $\lambda_{pp\mu}$ the transition rate from the atomic singlet state to the ortho p-μ-p molecular state. The ortho p-μ-p state further decays to the para p-μ-p molecular state. This rate is denoted by λ_{op}. Let $N_s(t)$, $N_{om}(t)$, and $N_{pm}(t)$ represent the numbers of muons at time t in the atomic singlet, ortho-molecular, and para-molecular states, respectively. They satisfy coupled kinetic equations, see Eq. (54a) in Ref. [21]. To integrate these coupled differential equations, we need to know the initial conditions.

For illustration purposes, let us consider a case in which there is one muon in the singlet state at $t = 0$; *i.e.*, $N_s(0) = 1$ and $N_{om}(0) = N_{pm}(0) = 0$. We then have

$$\begin{aligned} N_s(t) &= e^{-\lambda_2 t}, \quad N_{om}(t) = \frac{\lambda_{pp\mu}}{\lambda_2 - \lambda_3}(e^{-\lambda_3 t} - e^{-\lambda_2 t}), \\ N_{pm}(t) &= \frac{\lambda_{op}\lambda_{pp\mu}}{(\lambda_3 - \lambda_4)(\lambda_2 - \lambda_4)} e^{-\lambda_4 t} - \frac{\lambda_{op}\lambda_{pp\mu}}{(\lambda_2 - \lambda_3)(\lambda_3 - \lambda_4)} e^{-\lambda_3 t} \\ &\quad + \frac{\lambda_{op}\lambda_{pp\mu}}{(\lambda_2 - \lambda_3)(\lambda_2 - \lambda_4)} e^{-\lambda_2 t}, \end{aligned} \tag{5}$$

where $\lambda_2 = \lambda_0 + \lambda_{pp\mu} + \Lambda_s$, $\lambda_3 = \lambda_0 + \lambda_{op} + \Lambda_{op}$, $\lambda_4 = \lambda_0 + \Lambda_{pm}$. Here λ_0 is the muon natural decay rate, and Λ_{om}, and Λ_{pm} are the OMC rate in the ortho molecular, and para molecular states, respectively. The ortho- and para-molecular capture rates are given by

$$\Lambda_{om} = 2\gamma_O \left(\frac{3}{4}\Lambda_s + \frac{1}{4}\Lambda_t\right), \quad \Lambda_{pm} = 2\gamma_P \left(\frac{1}{4}\Lambda_s + \frac{3}{4}\Lambda_t\right), \tag{6}$$

where $2\gamma_O = 1.009$ and $2\gamma_P = 1.143$ [21]. Note that Eq.(6) is valid for both OMC and RMC.

At this point we discuss the numerical values of $\lambda_{pp\mu}$ and λ_{op}. The former shows a wide scatter in the literature, ranging from $\lambda_{pp\mu} = (1.89 \pm 0.20) \times 10^6$ s^{-1} to $(2.75 \pm 0.25) \times 10^6$ s^{-1} [23]. In this work, for the sake of definiteness, we employ the averaged value $\lambda_{pp\mu} = 2.5 \times 10^6$ s^{-1} (the main point of our argument is not affected by this choice). This value is comparable to the muon decay rate $\lambda_0 = 0.455 \times 10^6$ s^{-1}. As regards λ_{op}, there is a significant difference between the experimental and theoretical values; $\lambda_{op}^{exp} = (4.1 \pm 1.4) \times 10^4$ s^{-1} [7] as compared with $\lambda_{op}^{th} = (7.1 \pm 1.2) \times 10^4$ s^{-1} [21]. The dominant state for the OMC and RMC measurements is the ortho molecular state as is evident from the solution of Eq.(5). Furthermore, in the OMC experiment the time dependence of the population of each state plays an important role. In both the OMC and RMC experiments data taking starts at $t = t_i \neq 0$, and it is essential to incorporate this aspect into the A-L formula (see below).

3. Atom-Liquid (A-L) formula for OMC and RMC

In the OMC experiment (see Fig. 4 in Ref. [3]), μ^- beams arrive at the target on the average in $t_1 = 3$ μs-long burst with repetition rate 3000 Hz. The data collection typically starts 1 μs after the end of the 3 μs-long beam burst, and the measurement lasts until 306 μs after the end of the beam burst. As mentioned, the cascade processes leading to the μ-p ground state and the transition between the atomic hyperfine states are extremely fast. One therefore can safely ignore a time lag between the muon arrival time and the time at which the μ- p atomic hyperfine-singlet state is formed.

To proceed, we assume that the average time intervals of Ref.[3] cited above are actual time intervals. Then, provided all the muons arrive at the same time, we can choose with no ambiguity that arrival time as the origin of time ($t = 0$) and let $t = t_i$, the starting time for data collection, refer to that origin. However, the finite duration ($t_1 = 3$ μs) of the beam burst causes uncertainty in the value of $t = t_i$ to be used in Eq.(5), t_i can be anywhere between 1.0 μs and 4.0 μs. To account for this muon pulse duration time, t_1, we assume for simplicity that the beam pulse has a rectangular shape. Then, at time t the average number of residual muons are:

$$\bar{N}_\mu(t) \equiv \frac{1}{t_1}\int_0^{t_1} dt' N_\mu(t-t'). \tag{7}$$

where $N_\mu(t) = N_s(t) + N_{om}(t) + N_{pm}(t)$. The OMC experiment [3] counts the number of electrons produced by $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, and Λ_{liq} is deduced from the difference between the muon decay rate in liquid hydrogen and that in vacuum; the latter is determined from the number of positrons produced in $\mu^+ \to e^+ \nu_e \bar{\nu_\mu}$. We use the expression of Ref.[3] (and $t_i = 4$ μs)

$$\Lambda_{liq} \equiv \left(\frac{\int_{t_i}^\infty dt \frac{d\bar{N}_e}{dt}}{\int_{t_i}^\infty dt(t-t_i)\frac{d\bar{N}_e}{dt}}\right) - \lambda_0, \tag{8}$$

where $\bar{N}_e(t)$ is the averaged number of electrons produced at time t and $\frac{d\bar{N}_e(t)}{dt} = \lambda_0 \bar{N}_\mu(t)$. Here we have used the fact the duration of the measuring time (306 μs) is long enough to be treated as ∞.

On the other hand, for the RMC experiment [4, 5], the duration time of the beam burst is extremely short, 1 ns, and the data taking begins at $t_i = 365$ ns. We therefore can neglect the beam burst duration time in the RMC case, and we obtain

$$R_\gamma = \frac{N_\gamma(\infty) - N_\gamma(t_i)}{N_\mu(t_i)}. \tag{9}$$

Here $N_\gamma(t)$ is the number of photons obtained by integrating the photon spectrum over the interval, $60 \le E_\gamma \le 99$ MeV, and the production of photons in RMC is determined by

$$\frac{dN_\gamma(t)}{dt} = \Gamma_s N_s(t) + \Gamma_{om} N_{om}(t) + \Gamma_{pm} N_{pm}(t), \tag{10}$$

where $N_\gamma(0) = 0$ and where Γ_s, Γ_{om}, and Γ_{pm} are the theoretical RMC rates for the atomic singlet, ortho-, and para-molecular states, respectively.

4. Results and discussion

We estimate Λ_{liq} by using the atomic OMC rate obtained in our previous work within the framework of HBχPT up to NNLO [10]. Three A-L formulas are considered: Eq. (8), the A-L formula of Bardin *et al.*[7], and that of Bakalov *et al.*[21]. For the ortho-para transition rate we use either λ_{op}^{exp} or λ_{op}^{th}. We have obtained $\Lambda_{liq} \simeq 459$ s^{-1} with both Eq.(8) and Bardin *et al.*'s A-L formula, i.e. the two formulas agree with each other when we use λ_{op}^{exp}. However, if we use λ_{op}^{th}, the value of Λ_{liq} becomes significantly smaller ($\simeq$ 8%), illustrating the sensitivity of Λ_{liq} to λ_{op}. On the other hand, the use of Bakalov *et al.*'s A-L formula [21] gives too large a value for Λ_{liq}, regardless of whether we use λ_{op}^{exp} or λ_{op}^{th}. As mentioned before, the A-L formula of Bakalov *et al.*, which we employed in our previous work [10], corresponds to the choice $t_i = 0$, and this does not correspond to the experimental condition.

The estimate of R_γ is obtained with the use of the atomic RMC rates calculated using a phenomenological relativistic tree-level model [18], but with updated input parameters. The HBχPT calculations for RMC are found to agree well with the phenomenological approach [13, 14, 15, 24]. The result obtained with our A-L formula, Eq.(9) is in good agreement with that of Jonkmans *et al.*[4] provided we use λ_{op}^{exp}. In both cases, the predicted values of R_γ are significantly smaller than R_γ^{exp}; $R_\gamma^{exp}/R_\gamma^{th} \approx 1.5$. The use of λ_{op}^{th} instead of λ_{op}^{exp} enhances R_γ by about 9% but the increase is not large enough to reconcile R_γ^{th} with R_γ^{exp}. Thus it is not possible to reproduce R_γ^{exp} in the existing theoretical framework with the use of the standard set of input parameters.

Next, we discuss the sensitivity of Λ_{liq} and R_γ to possible changes in the values of g_P and the molecular mixing parameter ξ. In this discussion we use the phenomenological model of Fearing [18], a model which allows for the variation of the parameter g_P. As discussed by Weinberg [22], the ortho molecular p-μ-p spin 3/2 state, changes the molecular capture rates to

$$\Lambda'_{om} = \xi\Lambda_{om}(1/2) + (1-\xi)\Lambda_{om}(3/2), \tag{11}$$

where $\Lambda_{om}(1/2) = \Lambda_{om}$ of Eq.(6), and $\Lambda_{om}(3/2) = 2\gamma_O\Lambda_t$. (For RMC replace Λ with Γ.) Although the existing theoretical estimate suggests $\xi \simeq 1$[21, 25], we treat it here, as we did in Ref.[10], as a parameter to fit the data. In this phenomenological model, with the use of λ_{op}^{exp}, we can reproduce R_γ^{exp} by adopting either $g_P = 1.4g_P^{PCAC}$ or $\xi = 0.80$. However, with the same value λ_{op}, the OMC data require $g_P \leq 1.2g_P^{PCAC}$ or $\xi \geq 0.95$. Therefore, it is impossible to simultaneously fit the OMC and the RMC data by adjusting g_P and ξ. We also find that both the OMC and RMC capture rates are sensitive to the value of λ_{op}. If λ_{op} is taken to be smaller than $\lambda_{op}^{exp} = 4.1\times10^6$ s^{-1}, then it is not impossible to explain Λ_{liq}^{exp} and R_γ^{exp} within the phenomenological model with an increased value of g_P and $\xi \leq 0.95$. But we cannot attach too much significance to this possibility, since HBχPT constrains the value of g_P with high accuracy, and there is not much room left for adjusting g_P. The result of a more precise measurement of λ_{op} at TRIUMF [26] will shed much light on this issue.

Our findings are largely in the nature of reconfirming the conclusions stated in one way or another in the literature, but a coherent treatment of OMC and RMC in liquid hydrogen as described here is hoped to be useful. Our treatment is characterized by the use of the best available atomic capture rates obtained in HBχPT, and by an improved A-L formula. Although we have presented examples of simulation of the experimental conditions, they are only meant to serve illustrative purposes. Definitive analyses can be done only by the people who carried out the relevant experiments. Finally, we remark that a precise measurement of the OMC rate in hydrogen gas is planned at PSI[23]. This experiment would eliminate the ambiguity of the molecular transition rate discussed in this paper and directly test the HBχPT prediction [10, 15].

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